Kvadratické rovnice

Zadání

1) $x^2 - 6x - 216 = 0$

2) $8x - x^2 = 12$

3) $-x^2 + 5x = 0$

4) $x^2 - 9 = 0$

5) $4x^2 - 25 = 0$

6) $(x-6)^2 + (x-8)^2 = 100$

7) $x(x-2) + (x-2)(x+2) = 0$

8) $a^2 - 2a + 6 = 5(2-a)$

9) $(x-2)^2 + (x-9)^2 - (x-11)^2 = 0$

10) $3(2a-4) - (5-a)(2a+3) = 1$

11) $\frac{2}{3} \cdot \frac{x-2}{4} \cdot x = 1 - \frac{x}{6}$

12) $x^2 = x$

13) $\frac{1}{2x-4} + \frac{1-x}{x^2-2x} = \frac{1}{2}$

14) $\frac{1}{3x} - \frac{2}{x+2} = \frac{x}{x+2}$

15) $x + \frac{1}{x+1} = 1 + \frac{x}{x+1}$

16) $(2x+3)(3x-4) - (5-4x)(5x+6) = 10$

17) $\frac{1}{x+4} - \frac{4}{x-4} + \frac{x^2-20}{x^2-16} = 0$

18) $\frac{4x-15}{6x-13} - \frac{2x+12}{5x-12} = 8$

19) $\frac{-12}{2x-3} = 4 - \frac{6x^2+14}{10x-15}$

20) $\frac{5}{14-4x} - \frac{2x^2-5}{7-2x} = -\frac{8}{3} - \frac{5x+1}{21-6x}$

21) $\frac{x^2-5x+11}{x^2-7x+17} - \frac{5}{7} = 0$

22) $\frac{3}{x-5} - \frac{3-2x}{x^2-25} = -2$

Řešení

1)

$x^2 - 6x - 216 = 0 \quad a = 1; b = -6; c = -216$

$D = b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot (-216) = 36 + 864 = 900$

$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-6) + \sqrt{900}}{2 \cdot 1} = \frac{6 + 30}{2} = 18$

$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-6) - \sqrt{900}}{2 \cdot 1} = \frac{6 - 30}{2} = -12$

Řešení: $x_1 = 18; x_2 = -12$

2)

$8x - x^2 = 12$

$-x^2 + 8x - 12 = 0 \quad a = -1; b = 8; c = -12$

$D = b^2 - 4ac = 8^2 - 4 \cdot (-1) \cdot (-12) = 64 - 48 = 16$

$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-8 + \sqrt{16}}{2 \cdot (-1)} = \frac{-8 + 4}{-2} = 2$

$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-8 - \sqrt{16}}{2 \cdot (-1)} = \frac{-8 - 4}{-2} = 6$

Řešení: $x_1 = 2; x_2 = 6$

3)

$-x^2 + 5x = 0$

$x \cdot (-x + 5) = 0$

$x = 0 \quad nebo \quad -x + 5 = 0$

$x_1 = 0 \qquad\qquad -x = -5$

$\qquad\qquad\quad x_2 = 5$

Řešení: $x_1 = 0; x_2 = 5$

4)

$x^2 - 9 = 0$

$x^2 = 9$

$x_1 = +\sqrt{9} = +3$

$x_2 = -\sqrt{9} = -3$

Řešení: $x_1 = 3; x_2 = -3$

5)

$4x^2 - 25 = 0$

$4x^2 = 25 \quad / : 4$

$x^2 = \frac{25}{4}$

$x_1 = +\sqrt{\frac{25}{4}} = +\frac{5}{2}$

$x_2 = -\sqrt{\frac{25}{4}} = -\frac{5}{2}$

Řešení: $x_1 = \frac{5}{2};\ x_2 = -\frac{5}{2}$

6)

$(x-6)^2 + (x-8)^2 = 100$

$x^2 - 12x + 36 + x^2 - 16x + 64 - 100 = 0$

$2x^2 - 28x = 0$

$x \cdot (2x - 28) = 0$

$x = 0 \quad nebo \quad 2x - 28 = 0$

$x_1 = 0 \qquad\qquad 2x = 28$

$\qquad\qquad\quad x_2 = 14$

Řešení: $x_1 = 0;\ x_2 = 14$

7)

$x(x-2) + (x-2)(x+2) = 0$

$x^2 - 2x + x^2 + 2x - 2x - 4 = 0$

$2x^2 - 2x - 4 = 0 \quad a = 2; b = -2; c = -4$

$D = b^2 - 4ac = (-2)^2 - 4 \cdot 2 \cdot (-4) = 4 + 32 = 36$

$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-2) + \sqrt{36}}{2 \cdot 2} = \frac{2+6}{4} = 2$

$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-2) - \sqrt{36}}{2 \cdot 2} = \frac{2-6}{4} = -1$

Řešení: $x_1 = 2;\ x_2 = -1$

8)

$a^2 - 2a + 6 = 5(2 - a)$

$a^2 - 2a + 6 = 10 - 5a$

$a^2 - 2a + 5a + 6 - 10 = 0$

$a^2 + 3a - 4 = 0 \quad a = 1; b = 3; c = -4$

$D = b^2 - 4ac = 3^2 - 4 \cdot 1 \cdot (-4) = 9 + 16 = 25$

$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-3 + \sqrt{25}}{2 \cdot 1} = \frac{-3 + 5}{2} = 1$

$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-3 - \sqrt{25}}{2 \cdot 1} = \frac{-3 - 5}{2} = -4$

Řešení: $x_1 = 1;\ x_2 = -4$

9)

$(x - 2)^2 + (x - 9)^2 - (x - 11)^2 = 0$

$x^2 - 4x + 4 + x^2 - 18x + 81 - (x^2 - 22x + 121) = 0$

$2x^2 - 22x + 85 - x^2 + 22x - 121 = 0$

$x^2 - 36 = 0$

$x^2 = 36$

$x_1 = +\sqrt{36} = +6$

$x_2 = -\sqrt{36} = -6$

Řešení: $x_1 = 6;\ x_2 = -6$

10)

$3(2a - 4) - (5 - a)(2a + 3) = 1$

$6a - 12 - (10a + 15 - 2a^2 - 3a) - 1 = 0$

$6a - 12 - 10a - 15 + 2a^2 + 3a - 1 = 0$

$2a^2 - a - 28 = 0 \quad a = 2; b = -1; c = -28$

$D = b^2 - 4ac = (-1)^2 - 4 \cdot 2 \cdot (-28) = 1 + 224 = 225$

$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-1) + \sqrt{225}}{2 \cdot 2} = \frac{1 + 15}{4} = 4$

$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-1) - \sqrt{225}}{2 \cdot 2} = \frac{1 - 15}{4} = -\frac{14}{4} = -\frac{7}{2}$

Řešení: $x_1 = 4;\ x_2 = -\frac{7}{2}$

11)

$$\frac{2}{3}\cdot\frac{x-2}{4}\cdot x=1-\frac{x}{6}$$

$$\frac{2x(x-2)}{12}=1-\frac{x}{6}\ /\cdot 12$$

$$2x^2-4x=12-2x$$

$$2x^2-2x-12=0$$

$$D=(-2)^2-4\cdot 2\cdot(-12)=100$$

$$x_1=\frac{-(-2)+10}{4}=3$$

$$x_1=\frac{-(-2)-10}{4}=-2$$

Řešení: $x_1=3;\ x_2=-2$

12)

$$x^2=x$$

$$x^2-x=0$$

$$x(x-1)=0$$

$$x_1=0 \qquad x_2=1$$

Řešení: $x_1=0;\ x_2=1$

13)

$$\frac{1}{2x-4}+\frac{1-x}{x^2-2x}=\frac{1}{2}$$

$$\frac{1}{2(x-2)}+\frac{1-x}{x(x-2)}=\frac{1}{2}\ /\cdot 2x(x-2)$$

$$x+2(1-x)=x(x-2)$$

$$x+2-2x=x^2-2x$$

$$-x^2+x+2=0$$

$$D=1^2-4\cdot(-1)\cdot 2=9$$

$$x_1=\frac{-1+3}{2\cdot(-1)}=-1$$

$$x_2=\frac{-1-3}{2\cdot(-1)}=2$$

Podmínky: $x\neq 0;\ x\neq 2$

Řešení: $x=-1$

14)

$$\frac{1}{3x}-\frac{2}{x+2}=\frac{x}{x+2}\ /\cdot 3x(x+2)$$

$$x+2-3x\cdot 2=3x\cdot x$$

$$-3x^2-5x+2=0$$

$$D=(-5)^2-4\cdot(-3)\cdot 2=49$$

$$x_1=\frac{-(-5)+7}{2\cdot(-3)}=-2$$

$$x_1=\frac{-(-5)-7}{2\cdot(-3)}=\frac{1}{3}$$

Podmínky: $x\neq 0;\ x\neq -2$

Řešení: $x=\frac{1}{3}$

15)

$$x+\frac{1}{x+1}=1+\frac{x}{x+1}\ /\cdot(x+1)$$

$$x(x+1)+1=x+1+x$$

$$x^2+x+1=2x+1$$

$$x^2-x=0$$

$$x(x-1)=0$$

$x=0$ nebo $x-1=0$

$x=1$

Podmínky: $x\neq -1$

Řešení: $x_1=0;x_2=1$

16)

$$(2x+3)(3x-4)-(5-4x)(5x+6)=10$$

$$6x^2-8x+9x-12-(25x+30-20x^2-24x)=10$$

$$6x^2+x-12-25x-30+20x^2+24x=10$$

$$26x^2-52=0$$

$$26x^2=52\ /:26$$

$$x^2=2$$

$x_1=\sqrt{2}$ $x_2=-\sqrt{2}$

Řešení: $x_1=\sqrt{2};x_2=-\sqrt{2}$

17)

$$\frac{1}{x+4}-\frac{4}{x-4}+\frac{x^2-20}{x^2-16}=0 \ / \cdot (x+4)(x-4)$$

$$x-4-4(x+4)+x^2-20=0$$

$$x-4-4x-16+x^2-20=0$$

$$x^2-3x-40=0$$

$$D=b^2-4ac=(-3)^2-4\cdot 1\cdot(-40)=169$$

$$x_{12}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-3)\pm\sqrt{169}}{2\cdot 1}$$

$$x_1=8$$

$$x_2=-5$$

Podmínky: $x\neq\pm 4$

Řešení: $x_1=8; x_2=-5$

18)

$$\frac{4x-15}{6x-13}-\frac{2x+12}{5x-12}=8 \ / \cdot (6x-13)(5x-12)$$

$$(4x-15)(5x-12)-(2x+12)(6x-13)=8\cdot(6x-13)(5x-12)$$

$$20x^2-48x-75x+180-(12x^2-26x+72x-156)=8(30x^2-72x-65x+156)$$

$$20x^2-48x-75x+180-12x^2+26x-72x+156=240x^2-576x-520x+1248$$

$$-232x^2+927x-912=0$$

$$D=b^2-4ac=927^2-4\cdot(-232)\cdot(-912)=12993$$

$$x_{12}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-927\pm\sqrt{12993}}{2\cdot(-232)}$$

$$x_1\doteq 1,75$$

$$x_2\doteq 2,24$$

Podmínky: $x\neq\frac{13}{6}; x\neq\frac{12}{5}$

Řešení: $x_1\doteq 1,75; x_2\doteq 2,24$

19)

$$\frac{-12}{2x-3}=4-\frac{6x^2+14}{10x-15}$$

$$\frac{-12}{2x-3}=4-\frac{6x^2+14}{5(2x-3)}\ /\cdot 5(2x-3)$$

$$-60=20(2x-3)-1(6x^2+14)$$

$$-60=40x-60-6x^2-14$$

$$6x^2-40x+14=0$$

$$D=b^2-4ac=(-40)^2-4\cdot 6\cdot 14=1264$$

$$x_{12}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-40)\pm\sqrt{1264}}{2\cdot 6}$$

$$x_1\doteq 6,30$$

$$x_2\doteq 0,37$$

Podmínky: $x\neq\frac{3}{2}$

Řešení: $x_1\doteq 6,30; x_2\doteq 0,37$

20)

$$\frac{5}{14-4x}-\frac{2x^2-5}{7-2x}=-\frac{8}{3}-\frac{5x+1}{21-6x}$$

$$\frac{5}{2(7-2x)}-\frac{2x^2-5}{7-2x}=-\frac{8}{3}-\frac{5x+1}{3(7-2x)}\ /\cdot 6(7-2x)$$

$$15-6(2x^2-5)=-16(7-2x)-2(5x+1)$$

$$15-12x^2+30=-112+32x-10x-2$$

$$-12x^2-22x+159=0$$

$$D=b^2-4ac=(-22)^2-4\cdot(-12)\cdot 159=8116$$

$$x_{12}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-22)\pm\sqrt{8116}}{2\cdot(-12)}$$

$$x_1\doteq -4,67$$

$$x_2\doteq 2,84$$

Podmínky: $x\neq\frac{7}{2}$

Řešení: $x_1\doteq -4,67; x_2\doteq 2,84$

21)

$$\frac{x^2-5x+11}{x^2-7x+17}-\frac{5}{7}=0 \quad /\cdot 7\cdot\left(x^2-7x+17\right)$$

$$7\cdot\left(x^2-5x+11\right)-5\left(x^2-7x+17\right)=0$$

$$7x^2-35x+77-5x^2+35x-85=0$$

$$2x^2-8=0$$

$$x^2=4$$

$$x_1=2$$

$$x_2=-2$$

Podmínky: po dosazení obě čísla vyhovují.

Řešení: $x_1=2; x_2=-2$

22)

$$\frac{3}{x-5}-\frac{3-2x}{x^2-25}=-2$$

$$\frac{3}{x-5}-\frac{3-2x}{(x+5)(x-5)}=-2 \quad /\cdot(x+5)(x-5)$$

$$3(x+5)-1\cdot(3-2x)=-2(x+5)(x-5)$$

$$3x+15-3+2x=-2\left(x^2-25\right)$$

$$3x+15-3+2x=-2x^2+50$$

$$2x^2+5x-38=0$$

$$D=b^2-4ac=5^2-4\cdot 2\cdot(-38)=329$$

$$x_{12}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-5\pm\sqrt{329}}{2\cdot 2}$$

$$x_1\doteq 3,28$$

$$x_2\doteq -5,78$$

Podmínky: $x\neq\pm 5$

Řešení: $x_1\doteq 3,28; x_2\doteq -5,78$