Příklady, 1. část

Zadání

1) $x^2 - 6x - 216 = 0$

2) $8x - x^2 = 12$

3) $-x^2 + 5x = 0$

4) $x^2 - 9 = 0$

5) $4x^2 - 25 = 0$

6) $(x-6)^2 + (x-8)^2 = 100$

7) $x(x-2) + (x-2)(x+2) = 0$

8) $a^2 - 2a + 6 = 5(2-a)$

9) $(x-2)^2 + (x-9)^2 - (x-11)^2 = 0$

10) $3(2a-4) - (5-a)(2a+3) = 1$

11) $\dfrac{2}{3} \cdot \dfrac{x-2}{4} \cdot x = 1 - \dfrac{x}{6}$

12) $x^2 = x$

13) $\dfrac{1}{2x-4} + \dfrac{1-x}{x^2-2x} = \dfrac{1}{2}$

14) $\dfrac{1}{3x} - \dfrac{2}{x+2} = \dfrac{x}{x+2}$

Řešení

1)

$$x^2 - 6x - 216 = 0 \quad a = 1; b = -6; c = -216$$

$$D = b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot (-216) = 36 + 864 = 900$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-6) + \sqrt{900}}{2 \cdot 1} = \frac{6 + 30}{2} = 18$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-6) - \sqrt{900}}{2 \cdot 1} = \frac{6 - 30}{2} = -12$$

Řešení: $x_1 = 18; x_2 = -12$

2)

$$8x - x^2 = 12$$

$$-x^2 + 8x - 12 = 0 \quad a = -1; b = 8; c = -12$$

$$D = b^2 - 4ac = 8^2 - 4 \cdot (-1) \cdot (-12) = 64 - 48 = 16$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-8 + \sqrt{16}}{2 \cdot (-1)} = \frac{-8 + 4}{-2} = 2$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-8 - \sqrt{16}}{2 \cdot (-1)} = \frac{-8 - 4}{-2} = 6$$

Řešení: $x_1 = 2; x_2 = 6$

3)

$$-x^2 + 5x = 0$$

$$x \cdot (-x + 5) = 0$$

$$x = 0 \quad nebo \quad -x + 5 = 0$$

$$x_1 = 0 \qquad\qquad -x = -5$$

$$\qquad\qquad\qquad x_2 = 5$$

Řešení: $x_1 = 0; x_2 = 5$

4)

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x_1 = +\sqrt{9} = +3$$

$$x_2 = -\sqrt{9} = -3$$

Řešení: $x_1 = 3; x_2 = -3$

5)

$$4x^2 - 25 = 0$$

$$4x^2 = 25 \quad / : 4$$

$$x^2 = \frac{25}{4}$$

$$x_1 = +\sqrt{\frac{25}{4}} = +\frac{5}{2}$$

$$x_2 = -\sqrt{\frac{25}{4}} = -\frac{5}{2}$$

Řešení: $x_1 = \frac{5}{2}; \; x_2 = -\frac{5}{2}$

6)

$$(x-6)^2 + (x-8)^2 = 100$$

$$x^2 - 12x + 36 + x^2 - 16x + 64 - 100 = 0$$

$$2x^2 - 28x = 0$$

$$x \cdot (2x - 28) = 0$$

$$x = 0 \quad nebo \quad 2x - 28 = 0$$

$$x_1 = 0 \qquad\qquad 2x = 28$$

$$x_2 = 14$$

Řešení: $x_1 = 0; \; x_2 = 14$

7)

$$x(x-2) + (x-2)(x+2) = 0$$

$$x^2 - 2x + x^2 + 2x - 2x - 4 = 0$$

$$2x^2 - 2x - 4 = 0 \quad a = 2; b = -2; c = -4$$

$$D = b^2 - 4ac = (-2)^2 - 4 \cdot 2 \cdot (-4) = 4 + 32 = 36$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-2) + \sqrt{36}}{2 \cdot 2} = \frac{2+6}{4} = 2$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-2) - \sqrt{36}}{2 \cdot 2} = \frac{2-6}{4} = -1$$

Řešení: $x_1 = 2; \; x_2 = -1$

8)

$$a^2 - 2a + 6 = 5(2 - a)$$
$$a^2 - 2a + 6 = 10 - 5a$$
$$a^2 - 2a + 5a + 6 - 10 = 0$$
$$a^2 + 3a - 4 = 0 \quad a = 1; b = 3; c = -4$$
$$D = b^2 - 4ac = 3^2 - 4 \cdot 1 \cdot (-4) = 9 + 16 = 25$$
$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-3 + \sqrt{25}}{2 \cdot 1} = \frac{-3 + 5}{2} = 1$$
$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-3 - \sqrt{25}}{2 \cdot 1} = \frac{-3 - 5}{2} = -4$$

Řešení: $x_1 = 1$; $x_2 = -4$

9)

$$(x - 2)^2 + (x - 9)^2 - (x - 11)^2 = 0$$
$$x^2 - 4x + 4 + x^2 - 18x + 81 - (x^2 - 22x + 121) = 0$$
$$2x^2 - 22x + 85 - x^2 + 22x - 121 = 0$$
$$x^2 - 36 = 0$$
$$x^2 = 36$$
$$x_1 = +\sqrt{36} = +6$$
$$x_2 = -\sqrt{36} = -6$$

Řešení: $x_1 = 6$; $x_2 = -6$

10)

$$3(2a - 4) - (5 - a)(2a + 3) = 1$$
$$6a - 12 - (10a + 15 - 2a^2 - 3a) - 1 = 0$$
$$6a - 12 - 10a - 15 + 2a^2 + 3a - 1 = 0$$
$$2a^2 - a - 28 = 0 \quad a = 2; b = -1; c = -28$$
$$D = b^2 - 4ac = (-1)^2 - 4 \cdot 2 \cdot (-28) = 1 + 224 = 225$$
$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-1) + \sqrt{225}}{2 \cdot 2} = \frac{1 + 15}{4} = 4$$
$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-1) - \sqrt{225}}{2 \cdot 2} = \frac{1 - 15}{4} = -\frac{14}{4} = -\frac{7}{2}$$

Řešení: $x_1 = 4$; $x_2 = -\frac{7}{2}$

11)

$$\frac{2}{3} \cdot \frac{x-2}{4} \cdot x = 1 - \frac{x}{6}$$

$$\frac{2x(x-2)}{12} = 1 - \frac{x}{6} \quad / \cdot 12$$

$$2x^2 - 4x = 12 - 2x$$

$$2x^2 - 2x - 12 = 0$$

$$D = (-2)^2 - 4 \cdot 2 \cdot (-12) = 100$$

$$x_1 = \frac{-(-2)+10}{4} = 3$$

$$x_1 = \frac{-(-2)-10}{4} = -2$$

Řešení: $x_1 = 3;\ x_2 = -2$

12)

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x_1 = 0 \qquad x_2 = 1$$

Řešení: $x_1 = 0;\ x_2 = 1$

13)

$$\frac{1}{2x-4} + \frac{1-x}{x^2-2x} = \frac{1}{2}$$

$$\frac{1}{2(x-2)} + \frac{1-x}{x(x-2)} = \frac{1}{2} \quad / \cdot 2x(x-2)$$

$$x + 2(1-x) = x(x-2)$$

$$x + 2 - 2x = x^2 - 2x$$

$$-x^2 + x + 2 = 0$$

$$D = 1^2 - 4 \cdot (-1) \cdot 2 = 9$$

$$x_1 = \frac{-1+3}{2 \cdot (-1)} = -1$$

$$x_2 = \frac{-1-3}{2 \cdot (-1)} = 2$$

Podmínky: $x \neq 0;\ x \neq 2$
Řešení: $x = -1$

14)

$$\frac{1}{3x} - \frac{2}{x+2} = \frac{x}{x+2} \quad / \cdot 3x(x+2)$$

$$x + 2 - 3x \cdot 2 = 3x \cdot x$$

$$-3x^2 - 5x + 2 = 0$$

$$D = (-5)^2 - 4 \cdot (-3) \cdot 2 = 49$$

$$x_1 = \frac{-(-5)+7}{2 \cdot (-3)} = -2$$

$$x_1 = \frac{-(-5)-7}{2 \cdot (-3)} = \frac{1}{3}$$

Podmínky: $x \neq 0; x \neq -2$

Řešení: $x = \dfrac{1}{3}$